Swaap.finance: Introducing the Matrix-MM

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Abstract

Swaap.finance is a decentralized exchange (DEX) protocol providing simple, powerful financial products. We introduce here the Matrix Market Maker (MMM) system. It is a stochastic, asymmetric, oracle-guided, multi-asset constant geometric mean product market maker, and has been thought to address today's most faced problems in DEXes relying on liquidity providers, namely: impermanent loss, market risks, and asset management complexity. Its design allows to earn fees while getting Index ETF performances on arbitrary assets, in one single place, passively and with minimal risks. In the following, we cover the MMM's technical details and properties.

Keywords: Decentralized Finance, Matrix Market Maker, Index ETF

1 Introduction

With a strong 10× increase of its users over the last 18 months [1], DeFi protocols have been gaining traction in a spectacular manner. DEXes in particular are now operating $50+ billion worth of transactions on a monthly basis, and even surpassed Coinbase trading volume over the Q2 2021 [2].

Nevertheless, the space is still in its early development and some important hurdles remain both on a product side (*e.g.* overall complexity for non-specialists) as well as on the technical side (*e.g.* the so-called impermanent loss, market risks, capital inefficiency). This leave room for improvement and innovation, as those issues will need to be addressed in order to make the DeFi reach a mainstream adoption.

2 Background

In the AMM landscape, most popular protocols such as Uniswap [3], PancakeSwap [4], Balancer [5] or KyberSwap [6] rely on the constant geometric mean market maker (CGMMM) principle. This system implements a management policy which consists in a continuous percentage-of-portfolio driven rebalancing. This kind of portfolio strategy, labeled as "active" in the traditional finance world, actively tries to beat a certain index such as the global market.

While a controlled portfolio exposure *via* constant assets weighting certainly has appealing properties, such active strategies have historically never succeeded in beating the market in the long run.
Additionally, rebalancing is usually not free. In CGMMM systems, this process is particularly costly in most cases - *a.k.a* the impermanent loss - and it only makes them profitable investment vehicles in the scenario of weak-trend, variance-driven pools as described in [7] and [8].
However, these criteria have usually not been met and will not necessarily in the future outside of highly-correlated-assets pools (*e.g.*: stablecoin pools), considering their overall phenomenal yet heterogeneous past performances and the large value proposition differences these corresponding projects might have. Bitcoin and Ether are a good illustration of this phenomenon with a strong CAGR $> 150\%$ over the last 5 years but only a moderate price correlation over the last year.

Since then, other solutions have been proposed including Uniswap V3's concentrated liquidity principle [9], KyberSwap's Dynamic-MM [6] or DODOEx's Proactive-MM [10]. While these solutions all embed or permit a sort of market-aware pricing, we haven't found yet a solution both simple and capital efficient.

Consequently, the Matrix Market Maker was thought of as a passive way of investing, with market-level performance.

3 Matrix Market Maker

The MMM is a stochastic, asymmetric, oracle-guided, multi-asset constant geometric mean product market maker.

Notably, it has been designed to behave as a CGMMM when absorbing market inefficiencies, while being impermanent loss resistant thanks to it's oracle-guided pricing system.

Additionally, it embeds a stochastic spread mechanism to protect liquidity providers against market risks, particularly during high volatile market trends. It also ensures a high liquidity availability during less risky market conditions by charging competitive trading fees.

Furthermore, the MMM is multi-asset by nature and its dynamic weighting allows to fully capture the market performance as Index ETFs.

Finally, the MMM version presented here employs the CGMMM as price discovery scheme - as it has been found to be a strong and consistent incentive method for non-stable assets [7] - but it can be transposed to stablecoins or liquidity-optimized scenarios quite easily.

Here is the general terminology used in the following:

- j represents the input asset.
- i represents the output asset.
- w^i represents the initial weight of asset i
- r_i represents the reserve of asset i.
- $\delta_t^{i,j}$ represents the spread term of assets i, j as defined in equation 3
- $(\gamma_i)_t$ represents the dynamic weight of asset i at time t, as defined in equation 4
- $OP_t^{i,c}$ represents the oracle price of asset i in a given currency peg c, at time t.
- MP_t^i represents the true market price of asset i in a given currency peg c, at time t.
- SP_t^i represents the spot price of asset i in asset j terms, at time t.
- $r_{i,j}^{E_*}$ is the reserve of asset i such that $SP_t^{i,j} = OP_t^{i,j}$

3.1 Spot Price

We define SP_t the spot price matrix as follows:

$$((SP_t^{i,j})_{ij})_t = \left(\left(\begin{cases} \frac{r_j}{r_i} \cdot \frac{(\gamma_i)_t}{(\gamma_j)_t} \cdot \delta_t^{i,j} & \text{if } i \neq j \\ 1 & \text{else} \end{cases} \right)_{ij} \right)_t \tag{1}$$

3.1.1 δ: stochastic buy-sell spread

The general idea here is dual:

1. Provide to liquidity providers a dynamic form of coverage based on market conditions data such as asset pairs quote drift and volatility, in the fashion of the standard practice in traditional markets.

2. Increase the trading volume during less risky market conditions by charging competitive fees.

For this purpose we used the Geometric Brownian Motion to model the asset-pair quotation, which gives us the following spread term:

$$\delta_{\mathbf{t}}^{\mathbf{i,j}} = \mathbb{1}_{\{r_i \geq r_{i,j}^{E*}\}} + \mathbb{1}_{\{r_i < r_{i,j}^{E*}\}} \cdot \max\Big(1, (\mathbf{GBM}_{\mu,\sigma}^{i,j})_h\Big) \tag{2}$$

Where:

- h is a time horizon hyper-parameter
- $\mathbf{GBM}_{\mu,\sigma}^{\mathbf{i,j}}$ is a Geometric Brownian Motion stochastic process with initial value of 1 and the same drift μ and variance σ^2 as the asset i - asset j pair.

Then, we take the p-quantile of that random variable:

$$\delta_{\mathbf{t+h}}^{\mathbf{i,j}} = \mathbb{1}_{\{r_i \geq r_{i,j}^{E*}\}} + \mathbb{1}_{\{r_i < r_{i,j}^{E*}\}} \cdot \max\Big(1, \Phi^{-1}_{(\mathbf{GBM}_{\mu,\sigma}^{i,j})_h}(p)\Big) \tag{3}$$

Where:

- p is the coverage level hyper-parameter.
- Φ_X^{-1} is the inverse cumulative distribution function of the random variable X.

3.1.2 γ: dynamic weighting

The weights are designed to continuously adjust themselves in order to absorb the price moves.
This means that the costly, impermanent-loss-responsible price discovery action of arbitrageurs is totally removed from the equation, while their lucrative-for-the-pool role in the market inefficiencies correction is maintained.

In mathematical terms, our dynamic weights are defined as such:

$$(\gamma_i)_t = w^i \cdot \frac{OP_{t^*}^{c,i}}{OP_0^{c,i}} \tag{4}$$

3.2 Trading surface

To render this dynamic asymmetric system we define a family of parametric value functions V_t and its corresponding trading surface S_t.

$$V_t = \begin{cases} VS_t^{i,j} = r_i^{(\gamma_i)_t \cdot \delta_t^{i,j}} \cdot r_j^{(\gamma_j)_t} & \text{when } r_i < r_{i,j}^{E*} \\ VA_t^{i,j} = r_i^{(\gamma_i)_t} \cdot r_j^{(\gamma_j)_t} & \text{else} \end{cases} \tag{5}$$

3.3 Effective Price

The effective price $EP_t^{i,j}(\alpha)$ corresponds to the price in asset j terms for an amount $\alpha > 0$ of asset i, at time t.

We define it as follows:

$$EP_t^{i,j}(\alpha) = \int\limits_{P_\alpha} SP_t^{i,j}(p)dp \tag{6}$$

Where:

- P_α is the trading path $\{(x,y) \in [r_i, r_i - \alpha] \times [y, y + \beta_\alpha] | (x,y) \in S_t\}$

It can be shown (*c.f.* Appendix A) that:

$$EP_t^{i,j}(\alpha) = \begin{cases} EPS_0^i(r_i, \alpha) & \text{if } r_i < r_{i,j}^{E_*} \\ EPA_0^i(r_i, \alpha) & \text{if } r_i - \alpha > r_{i,j}^{E_*} \\ EPA_0^i(r_i, r_{i,j}^{E_*} - r_i) + EPS_0^i(r_{i,j}^{E_*}, \alpha - (r_{i,j}^{E_*} - r_i)) & \text{else} \end{cases} \tag{7}$$

Where $EPS_t^i(r_i, \alpha)$ (resp. $EPA_t^i(r_i, \alpha)$) represents the usual constant geometric mean market maker effective price in asset j terms for an amount $\alpha > 0$ of asset i at time t but defined on the shortage $VS_t^{i,j}$ (resp. abundance $VA_t^{i,j}$) surface, *i.e.*:

$$EPS_t^i(r_i, \alpha) = \left(\frac{VS_t^{ij}}{(r_i - \alpha)^{(\gamma_i)_t \cdot \delta_t^{i,j}}} \right)^{\frac{1}{(\gamma_j)_t}} - \left(\frac{VS_t^{i,j}}{r_i^{(\gamma_i)_t \cdot \delta_t^{i,j}}} \right)^{\frac{1}{(\gamma_j)_t}}$$

$$EPA_t^i(r_i, \alpha) = \left(\frac{VA_t^{i,j}}{(r_i - \alpha)^{(\gamma_i)_t}} \right)^{\frac{1}{(\gamma_j)_t}} - \left(\frac{VA_t^{i,j}}{r_i^{(\gamma_i)_t}} \right)^{\frac{1}{(\gamma_j)_t}}$$

3.4 Properties

3.4.1 Index Pool

Unlike traditional AMMs which adopt a continuous rebalancing strategy, MMM pools have, in addition to their fee-driven earnings, an asset-driven earnings stream that is able to capture the underlying assets' performance in its entirety. This is an immediate implication of the dynamic weighting policy as defined in equation 4.
As an example, in a $\left[\frac{\text{DAI}}{2}, \frac{\text{ETH}}{2}\right]$ MMM pool a $\times 2$ increase of ETH price will automatically drives the pool to a $\left[\frac{\text{DAI}}{3}, \frac{2\cdot\text{ETH}}{3}\right]$ weighting target, on the fly, and importantly: without impermanent loss.

Precisely, the asset i portfolio's value share relative to the asset j's one can be expressed as follows:,

$$(W^c_{i,j})_t = \frac{MP^{i,c}_t \cdot r_i}{MP^{j,c}_t \cdot r_j}$$

Under rational market hypotheses, the following equality is satisfied at equilibrium:

$$SP^{i,j}_t = \frac{MP^{i,c}_t}{MP^{j,c}_t} \Leftrightarrow \frac{r_j}{r_i} \cdot \frac{w^i_0}{w^j_0} \cdot \frac{\frac{OP^i_{t*}}{OP^i_0}}{\frac{OP^j_{t*}}{OP^j_0}} = \frac{MP^{i,c}_t}{MP^{j,c}_t}$$
$$\Leftrightarrow r_i \cdot MP^{i,c}_t = \frac{w^i_0}{w^j_0} \cdot \frac{\frac{OP^i_{t*}}{OP^i_0}}{\frac{OP^j_{t*}}{OP^j_0}} \cdot r_j \cdot MP^{j,c}_t$$

Hence, the asset i portfolio's value share, $\forall i$, can be defined in this way:

$$(W_i)_t = \frac{r_i \cdot MP^{i,c}_t}{TV^c_t} = \frac{w^i_0 \cdot \frac{OP^i_{t*}}{OP^i_0}}{\sum_{k=1}^{n} w^k_0 \cdot \frac{OP^k_{t*}}{OP^k_0}}$$

Where TV^c_t is the portfolio's total value in currency peg c, at time t.

We can notice that $(W_i)_t$ solely depends on both a) the initial weighting strategy and b) the asset i 's relative performance against the overall portfolio's.

Furthermore, as we expect $\frac{OP^i_{t*}}{MP^i_t}$ to be in practice consistently close to 1, we approximate r_i as follows:

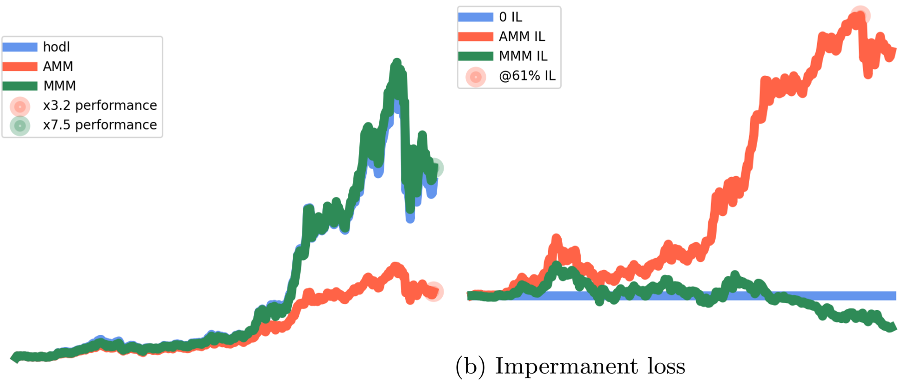
$$r_i \approx \frac{w^i_0}{w^j_0} \cdot \frac{OP^j_0}{OP^i_0} \cdot r_j \tag{8}$$

In this context, it essentially means 2 things for the pool:

1. There is no impermanent loss, by design.
2. Its performance $\neq$ compounding-fees + Index ETF. [1]

When simulating different portfolio strategies using real world data and under a *up to* 3% *market shift* hypothesis for the Matrix-MM pool - which is quite

[1] This holds as long as the price evolution remains equivalent to its market performance, as it has roughly been over the past years for most of the major assets. In case of assets with a singular monetary policy (for instance brutal changes in how asset is minted) the portfolio will just need a punctual reblancing in order to keep replicating the index. Impermanent loss resistance is not affected in any means.



(a) Raw performance without fees

(b) Impermanent loss
Negative loss corresponds to a profit.

Fig. 1: Properties of our 6-assets pool simulation

conservative in the light of Uniswap trading data - the MMM pool displays a strong and consistent ability to replicate the underlying index's performance. The raw portfolio value, *i.e.* without including fee earnings, and its impermanent loss are illustrated in figure 1, in the case of a 6-assets pool (DAI, ETH, BTC, ADA, BSC, LINK) on the [01/2020, 06/2021] period and with a 50% risky - 50% risk-free distribution.

3.4.2 Market risk coverage

In high volatility settings, Oracle based DEXes may be subject to losses. This is primarily due to:

1. An inability to keep the reserves balanced
2. Important prices fluctuations

When combined, and if the tokens in shortage appreciate in value, these factors can lead to permanent losses.

The stochastic spread mechanism has been designed to address such losses. It can be seen as an on-chain, dynamic, market-aware insurance: transactions contributing to an increase in risk for the pool are charged a premium proportionate to that risk.

This new stream of earnings directly adds up to the regular fees, and seamlessly compounds with the pool's total value, as illustrated in figure 2.
We show in Appendix B how this mechanism allows the pool to accumulate the premiums over time in an ever-growing fashion, as a coverage.

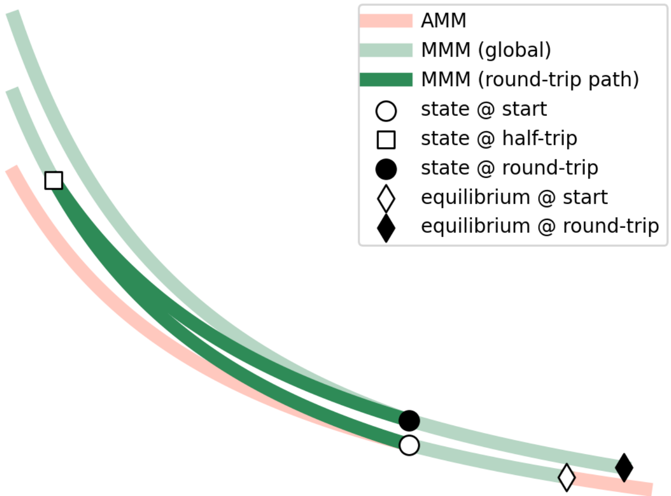


Fig. 2: MMM asymmetric pricing

4 Conclusion

Current LP-based DEX protocols rely on strategies and algorithms exhibiting serious technical limitations that will lead to losses for liquidity providers with high probability.

Hence, we suggested the introduction of a new stochastic, asymmetric and multi-asset form of MM system: the Matrix Market Maker.

Notably, it totally removes impermanent loss, and offers better-than-Index-ETF investment performances with minimal portfolio management, while covering liquidity providers from market risks, and thus, substantially lowering entry barriers to the DeFi space for non-specialists.

Appendix A Effective price: proof

Lemma 1

$$EP_t^{i,j}(\alpha) = \begin{cases} EPS_0^i(r_i,\alpha) & \text{if } r_i < r_{i,j}^{E_*} \\ EPA_0^i(r_i,\alpha) & \text{if } r_i - \alpha > r_{i,j}^{E_*} \\ EPA_0^i(r_i, r_{i,j}^{E_*} - r_i) + EPS_0^i(r_{i,j}^{E_*}, \alpha - (r_{i,j}^{E_*} - r_i)) & \text{else} \end{cases} \tag{A1}$$

With:

$$EPS_t^i(r_i,\alpha) = \left(\frac{VS_t^{ij}}{(r_i-\alpha)^{(\gamma_i)_t\cdot\delta_t^{i,j}}}\right)^{\frac{1}{(\gamma_j)_t}} - \left(\frac{VS_t^{i,j}}{r_i^{(\gamma_i)_t\cdot\delta_t^{i,j}}}\right)^{\frac{1}{(\gamma_i)_t}}$$

$$EPA_t^i(r_i,\alpha) = \left(\frac{VA_t^{i,j}}{(r_i-\alpha)^{(\gamma_i)_t}}\right)^{\frac{1}{(\gamma_j)_t}} - \left(\frac{VA_t^{i,j}}{r_i^{(\gamma_i)_t}}\right)^{\frac{1}{(\gamma_i)_t}}$$

Proof
We will only consider the $r_i < r_{i,j}^{E_*}$ case, as the proof for the 2 other cases is essentially the same.
Let $i \neq j$. As we integrate over P_α, we satisfy the constraints induced by the value function $VS_t^{i,j}$, so we know that:

$$\begin{aligned} VS_t^{i,j} &= (r_i-\alpha)^{(\gamma_i)_t\cdot\delta_t^{i,j}} \cdot (r_j+\beta_\alpha)^{(\gamma_j)_t} \\ \Leftrightarrow (r_j+\beta_\alpha) &= \left(\frac{V_t^{i,j}}{(r_i-\alpha)^{(\gamma_i)_t\cdot\delta_t^{i,j}}}\right)^{(\gamma_j)_t} \end{aligned}$$

Hence, we have:

$$\begin{aligned} \int\limits_{P_\alpha} SP_t^{i,j}(p)dp &= \int\limits_0^\alpha \frac{r_j+\beta_x}{r_i-x}\cdot\frac{(\gamma_i)_t}{(\gamma_j)_t}\cdot\delta_t^{i,j}dx \\ &= \int\limits_0^\alpha \left(\frac{VS_t^{i,j}}{(r_i-x)^{(\gamma_i)_t\cdot\delta_t^{i,j}}}\right)^{\frac{1}{(\gamma_j)_t}} \cdot \frac{1}{r_i-x}\cdot\frac{(\gamma_i)_t}{(\gamma_j)_t}\cdot\delta_t^{i,j}dx \\ &= F(\alpha) - F(0) \end{aligned}$$

Where F is the antiderivative, defined as such:

$$F(x) = \left(\frac{VS_t^{i,j}}{(r_i-x)^{(\gamma_i)_t\cdot\delta_t^{i,j}}}\right)^{\frac{1}{(\gamma_j)_t}} \tag{A2}$$

□

Appendix B Premium accumulation: proof

Lemma 2 We can show that any round trip asset $i \Leftrightarrow$ asset j will lead to an increase in the pool's total value.

Proof
Let's consider r_i, r_j the initial reserves of assets i and j such that the price of asset i is $> MP^{i,j}$ the market price. By definition, we are in a shortage of asset i with respect to asset j, and so a premium is applied when buying asset i with asset j, this premium being represented by the stochastic spread term in equation A1. Now, let's consider as well that the market conditions imply a strictly positive spread term, which corresponds to real world market conditions.

In the following, we will drop the time index t as we consider the market and oracle prices to be fixed during this round-trip trade.

Let α_1 be the quantity of asset i we will buy in exchange of β_1 asset j. Let α_2 and β_2 be the quantities for the corresponding inverse operations.
As this is a round-trip trade, $\alpha_1 = \alpha_2 = \alpha$.
We will show that we will end with the exact same amount of asset i but with a $\bar{\bar{r}}_j - r_j > 0$ excess of asset j.

Based on the MMM equation, when buying $\alpha > 0$ quantity of asset i for $\beta_1 > 0$ quantity of asset j under shortage conditions, we move on this curve:

$$(r_i - \alpha)^{\gamma_i \cdot \delta_{ij}} \cdot (r_j + \beta_1)^{\gamma_j} = r_i^{\gamma_i \cdot \delta_{ij}} \cdot r_j^{\gamma_j}$$

Which gives us the following:

$$\begin{aligned}
\bar{r}_j &= (r_j + \beta_1) \\
&= \left(\frac{r_i^{\gamma_i \cdot \delta_{ij}} \cdot r_j^{\gamma_j}}{(r_i - \alpha)^{\gamma_i \cdot \delta_{ij}}} \right)^{\frac{1}{\gamma_j}} \\
&= r_j \cdot \left(\frac{r_i^{\gamma_i}}{(r_i - \alpha)^{\gamma_i}} \right)^{\frac{\delta_{ij}}{\gamma_j}}
\end{aligned}$$

Then, when selling back the amount α of asset i for $\beta_2 > 0$ asset j we are not subject to the stochastic spread anymore, so we now move on that curve:

$$(r_i - \alpha + \alpha)^{\gamma_i} \cdot (\bar{r}_j - \beta_2)^{\gamma_j} = (r_i - \alpha)^{\gamma_i} \cdot r_j^{\gamma_j}$$

We can then further develop:

$$\begin{aligned}
\bar{\bar{r}}_j &= (\bar{r}_j - \beta_2) \\
&= \left(\frac{(r_i - \alpha)^{\gamma_i} \cdot \bar{r}_j^{\gamma_j}}{r_i^{\gamma_i}} \right)^{\frac{1}{\gamma_j}} \\
&= \bar{r}_j \cdot \left(\frac{(r_i - \alpha)^{\gamma_i}}{r_i^{\gamma_i}} \right)^{\frac{1}{\gamma_j}} \\
&= r_j \cdot \left(\frac{r_i^{\gamma_i}}{(r_i - \alpha)^{\gamma_i}} \right)^{\frac{1}{\gamma_j} \cdot (\delta_{ij} - 1)}
\end{aligned}$$

Since $\delta_{ij} > 1$ and $\alpha > 0$, we can conclude:

$$\bar{\bar{r}}_j > r_j$$

$\square$

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